

Floating Point Routines for the 6502

by Roy Rankin, Department of Mechanical Engineering,
Stanford University, Stanford, CA 94305
(415) 497-1822

and

Steve Wozniak, Apple Computer Company
770 Welch Road, Suite 154
Palo Alto, CA 94304
(415) 326-4248

Editor's Note: Although these routines are for the 6502, it would appear that one could generate equivalent routines for most of the "traditional" microprocessors, relatively easily, by following the flow of the algorithms given in the excellent comments included in the program listing. This is particularly true of the transcendental functions which were directly modeled after well-known and proven algorithms, and for which, the comments are relatively machine-independent.

These floating point routines allow 6502 users to perform most of the more popular and desired floating point and transcendental functions, namely:

- Natural Log - LOG
- Common Log - LOG10
- Exponential - EXP
- Floating Add - FADD
- Floating Subtract - FSUB
- Floating Multiply - FMUL
- Floating Divide - FDIV
- Convert Floating to Fixed - FIX
- Convert Fixed to Floating - FLOAT

They presume a four-byte floating point operand consisting of a one-byte exponent ranging from -218 through +127, and a 24-bit two's complement mantissa between 1.0 and 2.0.

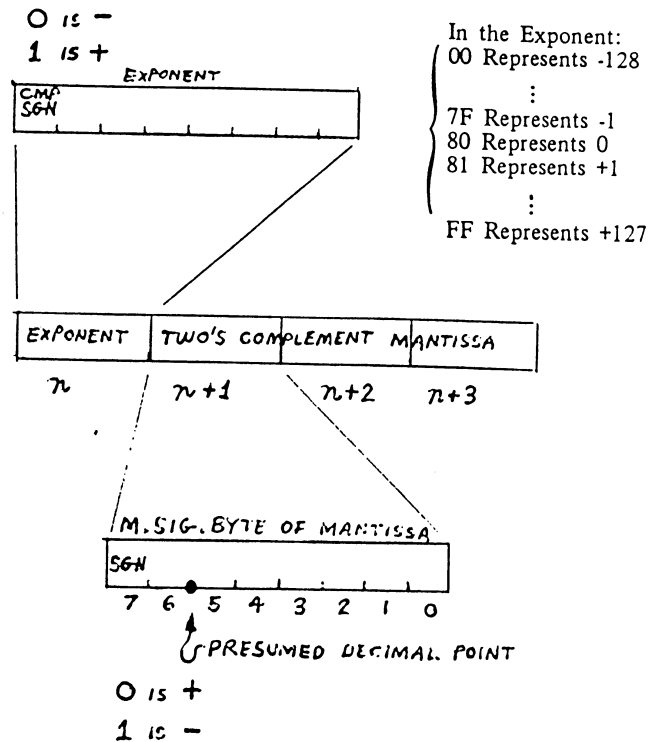
The floating point routines were done by Steve Wozniak, one of the principals in Apple Computer Company. The transcendental functions were patterned after those offered by Hewlett-Packard for their HP2100 minicomputer (with some modifications), and were done by Roy Rankin, a Ph.D. student at Stanford University.

There are three error traps; two for overflow, and one for prohibited logarithm argument. ERROR (1D06) is the error exit used in event of a non-positive log argument. OVFLW (1E3B) is the error exit for overflow occurring during calculation of e to some power. OVFL (1FE4) is the error exit for overflow in all of the floating point routines. There is no trap for underflow; in such cases, the result is set to 0.0.

All routines are called and exited in a uniform manner: The argument(s) are placed in the specified floating point storage locations (for specifics, see documentation preceding each routine in the listing), then a JSR is used to enter the desired routine. Upon normal completion, the called routine is exited via a subroutine return instruction (RTS).

Note: The preceding documentation was written by the Editor, based on phone conversations with Roy and studying the listing. There is a high probability that it is correct. However, since it was not written nor reviewed by the authors of these routines, the preceding documentation may contain errors in concept or in detail.

- JCW, Jr.



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BASIC FLOATING POINT ROUTINES
FOR 6502 MICROPROCESSOR
BY R. RANKIN AND S. WOZNIAK

CONSISTING OF:
NATURAL LOG
COMMON LOG
EXPONENTIAL (E**X)
FLOAT FIX
FADD FSUB
FMUL FDIV

FLOATING POINT REPRESENTATION (4-BYTES)
EXHONENT BYTE 1
MANTISSA BYTES 2-4

MANTISSA: TWO'S COMPLEMENT REPRESENTATION WITH SIGN IN MSB OF HIGH-ORDER BYTE. MANTISSA IS NORMALIZED WITH AN ASSUMED DECIMAL POINT BETWEEN BITS 5 AND 6 OF THE HIGH-ORDER BYTE. THUS THE MANTISSA IS IN THE RANGE 1. TO 2. EXCEPT WHEN THE NUMBER IS LESS THAN 2**(-128).

EXHONENT: THE EXHONENT REPRESENTS POWERS OF TWO. THE REPRESENTATION IS 2'S COMPLEMENT EXCEPT THAT THE SIGN BIT (BIT 7) IS COMPLEMENTED. THIS ALLOWS DIRECT COMPARISON OF EXHONENTS FOR SIZE SINCE THEY ARE STORED IN INCREASING NUMERICAL SEQUENCE RANGING FROM 100 (-128) TO 1FF (+127) (\$ MEANS NUMBER IS HEXADECIMAL).

REPRESENTATION OF DECIMAL NUMBERS: THE PRESENT FLOATING POINT REPRESENTATION ALLOWS DECIMAL NUMBERS IN THE APPROXIMATE RANGE OF 10**(-33) THROUGH 10**(33) WITH 6 TO 7 SIGNIFICANT DIGITS.

0003	EA	SGN	ORG 3	SET BASE PAGE ADDRESSES
0004	EA	NO	NOP	
0005	00 00 00	M2	BSS 3	EXHONENT 2
0006	EA	M1	NOP	MANTISSA 2
0007	00 00 00	M1	BSS 3	EXHONENT 1
0008	E	BSS 4		MANTISSA 1
0009	Z	BSS 4		SCRATCH
0010	T	BSS 4		

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0010      SEXP  BSS 4
001C 00      INT  BSS 1
          *
1D00      *      ORG $1D00      STARTING LOCATION FOR LOG
          *
          *      NATURAL LOG OF MANT/EXP1 WITH RESULT IN MANT/EXP1
          *
1D00 A5 09      LOG      LDA M1
1D02 F0 02      BEQ ERROR
1D04 10 01      BPL CONT      IF ARG>0 OK
1D06 00      BRK      ERROR ARG<=0
          *
1D07 20 1C 1F  CONT      JSR SWAP      MOVE ARG TO EXP/TRANS2
1D08 A5 04      LDA X2      HOLD EXPONENT
1D0C A0 00      LDY +$80
1D0E 04 04      STY X2      SET EXPONENT 2 TO 0 ($80)
1D10 49 00      EOR +$80      COMPLIMENT SIGN BIT OF ORIGINAL EXPONENT
1D12 05 0A      STA M1+1      SET EXPONENT INTO MANTISSA 1 FOR FLOAT
1D14 A9 00      LDA +0
1D16 05 09      STA M1      CLEAR MSB OF MANTISSA 1
1D18 20 2C 1F  JSR FLOAT      CONVERT TO FLOATING POINT
1D1A 02 03      LDX +3      4 BYTE TRANSFERS
1D1D 05 04      SEXP1      LDA X2,X
1D1F 95 10      STA Z,X      COPY MANTISSA TO Z
1D21 05 00      LDA X1,X
1D23 95 10      STA SEXP,X      SAVE EXPONENT IN SEXP
1D25 0D D1 1D  LDA R22,X      LOAD EXP/TRANS1 WITH SORT(2)
1D28 95 00      STA X1,X
1D2A CA      DEX
1D2B 10 F0      BPL SEXP1
1D2D 20 4A 1F  JSR FSUB      Z-SORT(2)
1D30 A2 03      LDX +3      4 BYTE TRANSFER
1D32 05 00      SAVET      LDA X1,X      SAVE EXP/TRANS1 AS T
1D34 95 14      STA T,X
1D36 05 10      LDA Z,X      LOAD EXP/TRANS1 WITH Z
1D38 95 00      STA X1,X
1D3A 0D D1 1D  LDA R22,X      LOAD EXP/TRANS2 WITH SORT(2)
1D3D 95 04      STA X2,X
1D3F CA      DEX
1D40 10 F0      BPL SAVET
1D42 20 50 1F  JSR FADD      Z+SORT(2)
1D45 A2 03      LDX +3      4 BYTE TRANSFER
1D47 05 14      TH2      LDA T,X
1D49 95 04      STA X2,X      LOAD T INTO EXP/TRANS2
1D4B CA      DEX
1D4C 10 F9      BPL TH2
1D4E 20 9D 1F  JSR FDIV      T*(Z-SORT(2))/(Z+SORT(2))
1D51 A2 03      LDX +3      4 BYTE TRANSFER
1D53 05 00      MIT      LDA X1,X
1D55 95 14      STA T,X      COPY EXP/TRANS1 TO T AND
1D57 95 04      STA X2,X      LOAD EXP/TRANS2 WITH T
1D59 CA      DEX
1D5A 10 F7      BPL MIT
1D5C 20 77 1F  JSR FMUL      T*T
1D5F 20 1C 1F  JSR SWAP      MOVE T*T TO EXP/TRANS2
1D62 A2 03      LDX +3      4 BYTE TRANSFER
1D64 0D E1 1D  MIC      LDA C,X
1D67 95 00      STA X1,X      LOAD EXP/TRANS1 WITH C
1D69 CA      DEX
1D6A 10 F8      BPL MIC
1D6C 20 4A 1F  JSR FSUB      T*T-C
1D6F A2 03      LDX +3      4 BYTE TRANSFER
1D71 0D DD 1D  M2MB      LDA MB,X
1D74 95 04      STA X2,X      LOAD EXP/TRANS2 WITH MB
1D76 CA      DEX
1D77 10 F8      BPL M2MB
1D79 20 9D 1F  JSR FDIV      MB/(T*T-C)
1D7C A2 03      LDX +3      4 BYTE TRANSFER
1D7E 0D D9 1D  M2A1      LDA A1,X
1D81 95 04      STA X2,X      LOAD EXP/TRANS2 WITH A1
1D83 CA      DEX
1D84 10 F8      BPL M2A1
1D86 20 50 1F  JSR FADD      MB/(T*T-C)+A1
1D89 A2 03      LDX +3      4 BYTE TRANSFER
1D8B 05 14      M2T      LDA T,X
1D8D 95 04      STA X2,X      LOAD EXP/TRANS2 WITH T
1D8F CA      DEX
1D90 10 F9      BPL M2T
1D92 20 77 1F  JSR FMUL      (MB/(T*T-C)+A1)*T
1D95 A2 03      LDX +3      4 BYTE TRANSFER
1D97 0D E5 1D  M2ML      LDA MHLF,X
1D9A 95 04      STA X2,X      LOAD EXP/TRANS2 WITH MHLF (.5)
1D9C CA      DEX
1D9D 10 F8      BPL M2ML
1D9F 20 50 1F  JSR FADD      +.5
1DA2 A2 03      LDX +3      4 BYTE TRANSFER
1DA4 05 10      LDEXP      LDA SEXP,X
1DA6 95 04      STA X2,X      LOAD EXP/TRANS2 WITH ORIGINAL EXPONENT
1DA8 CA      DEX
1DA9 10 F9      BPL LDEXP
1DAB 20 50 1F  JSR FADD      +EXPN
1DAE A2 03      LDX +3      4 BYTE TRANSFER
1D80 0D D5 1D  MLE2      LDA LE2,X
1DB3 95 04      STA X2,X      LOAD EXP/TRANS2 WITH LN(2)
1DB5 CA      DEX
1DB6 10 F8      BPL MLE2
1DB8 20 77 1F  JSR FMUL      *LN(2)
1DBB 60      RTS      RETURN RESULT IN MANT/EXP1
          *
          *      COMMON LOG OF MANT/EXP1 RESULT IN MANT/EXP1
          *
1DBC 20 00 1D  LOG10      JSR LOG      COMPUTE NATURAL LOG
1DBF A2 03      LDX +3

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1DC1 8D CD 1D  L10      LDA LN10,X
1DC4 95 04      STA X2,X      LOAD EXP/TRANS2 WITH 1/LN(10)
1DC6 CA      DEX
1DC7 10 F8      BPL L10
1DC9 20 77 1F  JSR FMUL      LOG10(X)*LN(X)/LN(10)
1DCC 60      RTS
          *
1DCD 7E 6F      LN10      DCM 0.4342945
1DD1 80 5A      R22      DCM 1.4142136      SORT(2)
1DD5 7F 58      LE2      DCM 0.69314718      LOG BASE E OF 2
1DD9 80 52      A1      DCM 1.2920074
1DDD 81 A8      MB      DCM -2.6390577
1DE1 80 6A      C      DCM 1.6567626
1DE5 7F 48      MHLF      DCM 0.5
          *
1E00      *      ORG $1E00      STARTING LOCATION FOR EXP
          *
          *      EXP OF MANT/EXP1 RESULT IN MANT/EXP1
          *
1E00 A2 03      EXP      LDX +3      4 BYTE TRANSFER
1E02 8D D8 1E  LDA LZE,X
1E05 95 04      STA X2,X      LOAD EXP/TRANS2 WITH LOG BASE 2 OF E
1E07 CA      DEX
1E08 10 F8      BPL EXP+2
1E0A 20 77 1F  JSR FMUL      LOG2(E)*X
1E0D A2 03      LDX +3      4 BYTE TRANSFER
1E0F 05 00      FSA      LDA X1,X
1E11 95 10      STA Z,X      STORE EXP/TRANS1 IN Z
1E13 CA      DEX
1E14 10 F9      BPL FSA
1E16 20 E8 1F  JSR FIX      SAVE Z=LN(2)*X
1E19 A5 0A      LDA M1+1      CONVERT CONTENTS OF EXP/TRANS1 TO AN INTEGER
1E1B 05 1C      STA INT
1E1D 30      SEC
1E1E E9 7C      SBC +124      SAVE RESULT AS INT
1E20 A5 09      LDA M1      SET CARRY FOR SUBTRACTION
1E22 E9 08      SBC +0      INT-124
1E24 10 15      BPL OVFLW      OVERFLOW INT>124
1E26 10      CLC      CLEAR CARRY FOR ADD
1E27 A5 0A      LDA M1+1
1E29 69 78      ADC +120      ADD 120 TO INT
1E2B A5 09      LDA M1
1E2D 05 00      ADC +0
1E2F 10 00      BPL CONTIN      IF RESULT POSITIVE CONTINUE
1E31 A9 00      LDA +0      INT<120 SET RESULT TO ZERO AND RETURN
1E33 A2 03      LDX +3      4 BYTE MOVE
1E35 95 00      ZERO      STA X1,X      SET EXP/TRANS1 TO ZERO
1E37 CA      DEX
1E38 10 F8      BPL ZERO
1E3A 60      RTS      RETURN
          *
1E3B 00      *      OVFLW      BRK      OVERFLOW
          *
1E3C 20 2C 1F  CONTIN      JSR FLOAT      FLOAT INT
1E3F A2 03      LDX +3
1E41 05 10      ENTD      LDA Z,X
1E43 95 04      STA X2,X      LOAD EXP/TRANS2 WITH Z
1E45 CA      DEX
1E46 10 F9      BPL ENTD
1E48 20 4A 1F  JSR FSUB      Z-Z-FLOAT(INT)
1E4B A2 03      LDX +3      4 BYTE MOVE
1E4D 05 00      ZSAV      LDA X1,X
1E4F 95 10      STA Z,X      SAVE EXP/TRANS1 IN Z
1E51 95 04      STA X2,X      COPY EXP/TRANS1 TO EXP/TRANS2
1E53 CA      DEX
1E54 10 F7      BPL ZSAV
1E56 20 77 1F  JSR FMUL      Z*Z
1E59 A2 03      LDX +3      4 BYTE MOVE
1E5B 0D DC 1E  LA2      LDA A2,X
1E5E 95 04      STA X2,X      LOAD EXP/TRANS2 WITH A2
1E60 05 00      LDA X1,X
1E62 95 10      STA SEXP,X      SAVE EXP/TRANS1 AS SEXP
1E64 CA      DEX
1E65 10 F4      BPL LA2
1E67 20 50 1F  JSR FADD      Z*Z+A2
1E6A A2 03      LDX +3      4 BYTE MOVE
1E6C 0D E8 1E  LB2      LDA B2,X
1E6F 95 04      STA X2,X      LOAD EXP/TRANS2 WITH B2
1E71 CA      DEX
1E72 10 F8      BPL LB2
1E74 20 9D 1F  JSR FDIV      T*B2/(Z*Z+A2)
1E77 A2 03      LDX +3      4 BYTE MOVE
1E79 05 00      DLOAD      LDA X1,X
1E7B 95 14      STA T,X      SAVE EXP/TRANS1 AS T
1E7D 0D E4 1E  *      LDA C2,X
1E80 95 00      STA X1,X      LOAD EXP/TRANS1 WITH C2
1E82 05 10      LDA SEXP,X
1E84 95 04      STA X2,X      LOAD EXP/TRANS2 WITH SEXP
1E86 CA      DEX
1E87 10 F8      BPL DLOAD
1E89 20 77 1F  JSR FMUL      Z*Z=C2
1E8B 20 1C 1F  JSR SWAP      MOVE EXP/TRANS1 TO EXP/TRANS2
1E8F A2 03      LDX +3      4 BYTE TRANSFER
1E91 05 14      LTHP      LDA T,X
1E93 95 00      STA X1,X      LOAD EXP/TRANS1 WITH T
1E95 CA      DEX

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[illegible]